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Electric charge quantization without anomalies?

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Abstract

In gauge theories like the standard model, the electric charges of the fermions can be heavily constrained from the classical structure of the theory and from the cancellation of anomalies. We argue that the anomaly conditions are not quite as well motivated as the classical constraints, since it is possible that new fermions could exist which cancel potential anomalies. For this reason we examine the classically allowed electric charges of the known fermions and we point out that the electric charge of the tau neutrino is classically allowed to be non-zero. The experimental bound on the electric charge of the tau neutrino is many orders of magnitude weaker than for any other known neutrino. We discuss possible modifications of the minimal standard model such that electric charge is quantized classically.

The quantization of the electric charges of most of the known fermions is a well established experimental phenomenon. An approach to a theoretical understanding of this phenomenon has emerged in recent years based on the standard model [1]. The standard model is a gauge theory with gauge group

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y, \tag{1}$$

which is assumed to be spontaneously broken by the vacuum expectation value (VEV) of a scalar doublet $\phi \sim (1, 2, 1)$ (whose $U(1)_Y$ charge can be normalized to 1 without loss of generality due to a scaling symmetry; $g \to \eta g, Y \to Y/\eta$, where g is the $U(1)_Y$ coupling constant, and Y is the generator of the $U(1)_Y$ gauge group.). The gauge symmetry of the Lagrangian can be used to choose the standard form for the vacuum:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{2}$$

The VEV of ϕ breaks $SU(2)_L \otimes U(1)_Y$ (but does'nt break $SU(3)_c$ of course) leaving an unbroken $U(1)$ symmetry, $U(1)_Q$, which is identified with electromagnetism and its generator Q is the linear combination (which annihilates the VEV of eq.(2)):

$$Q = I_3 + Y/2. \tag{3}$$

(The normalization of Q is undetermined, and we have taken the convention of normalizing it so that the charged W bosons will have charge 1.)

There are two quite distinct ways in which the standard model constrains the electric charges of the fermions. Firstly, there are a set of constraints which follow from the consistency of the theory at the classical level (such as the requirement that the Lagrangian be gauge invariant), while there are other constraints which follow from the consistency of the theory at the quantum level (i.e. the anomaly cancellation conditions). We first discuss the classical constraints. The invariance of the Yukawa Lagrangian (or equivalently, the electromagnetic invariance of the fermion mass terms and the quark flavour mixing terms in the weak interaction) constrains the electric charges. For example, the electron mass term in the lagrangian is invariant under $U(1)_Q$ if and only if the electric charge of the left-handed electron is equal to the charge of the right-handed electron. A similiar arguement holds for the quarks, so that for one generation, there are four electric charges, the charge of the electron, the charge of the neutrino, the charge of the up quark and the charge of the down quark [2]. A second piece of information about the fermion electric charges can be obtained by observing that the left-handed fermions are in $SU(2)_L$ doublets. Since $SU(2)_L$ and $U(1)_Y$ are a direct product (i.e act independently of each other), the members of the $SU(2)_L$ doublet have the same $U(1)_Y$ charge and thus the difference of the electric charges of the members of the $SU(2)_L$ doublet is just the difference of their I_3 eigenvalue (which is just equal to 1 with our normalization). Hence we have the information that the electric charge of the electron neutrino minus

the electric charge of the electron is 1, and the electric charge of the up quark minus the electric charge of the down quark is equal to 1. So in each generation there are only two unknown electric charges, which can be taken as the electric charge of the neutrino and the electric charge of the down quark.

Since the CKM matrix is non-diagonal, there are additional classical constraints from the flavour mixing terms in the standard model Lagrangian. For example, the W boson couples a u quark to a s quark, as well as a u quark to a d quark. Since the Lagrangian is invariant under $U(1)_Q$, the existence of these terms tells us that the s and d quark electric charges are equal. A similiar mixing happens of course with the third generation quarks, so that the electric charge of the b quark must be equal to the electric charges of the s and d quarks. Hence each of the three known generations of quarks have exactly the same electric charges. No such mixing has been observed in the lepton sector, and in the minimal standard model there can be no such mixing as the neutrinos are massless in that model. Hence the constraints from mass and mixing together with the $SU(2)_L$ doublet structure of the left-handed fermions tell us that there are *four* classically undetermined electric charges in the standard model. These four undertermined electric charges can be taken to be the electric charges of the three neutrinos and the down quark (we denote these four electric charges as $Q(\nu_e), Q(\nu_\mu), Q(\nu_\tau)$, and $Q(d)$). All of the other fermion electric charges can be uniquely determined in terms of these four classically undetermined parameters. Experimentally, it is known that

$$\begin{aligned}
Q(d) &= -1/3 \pm \delta_d,\ \delta_d < 10^{-21} \\
Q(\nu_e) &= 0 \pm \delta_{\nu_e},\ \delta_{\nu_e} < 10^{-21} \\
Q(\nu_\mu) &= 0 \pm \delta_{\nu_\mu},\ \delta_{\nu_\mu} < 10^{-9} \\
Q(\nu_\tau) &= 0 \pm \delta_{\nu_\tau},\ \delta_{\nu_\tau} < \ 3 \times 10^{-4}
\end{aligned} \tag{4}$$

where the experimental bounds (i.e the delta parameters) come from experiments on the neutron charge [3], experiments on the neutrality of matter [4], experiments on $\nu_\mu e$ scattering [5]. The experimental bound on the electric charge of the ν_τ has not been specifically studied previously (as far as we are aware) and the constraint given in Eq.(4) comes from an analysis in Ref.[6] which examines the experimental bounds on the electric charge of a hypothetical exotic "mini-charged" particle [7].

At this stage one can argue that further constraints can be obtained by assuming that gauge anomalies cancel. Anomalies imply the loss of a classical symmetry in the quantum theory [8]. If we assume that triangle anomalies cancel then we have two constraints which are not independent of the classical constraints. They are the $[U(1)_Q]^3$ and $[SU(2)]^2U(1)_Q$ anomaly condition. The cancellation of the $[U(1)_Q]^3$ anomaly gives the constraint:

$$Q(\nu_e)^3 + Q(\nu_\mu)^3 + Q(\nu_\tau)^3 = 0. \tag{5}$$

The equation only involves the neutrinos, since there is no contribution from the charged fermions (this is because the classical constraints derived from the existence of non-zero masses for the charged fermions implies that $U(1)_Q$ is vector like for the charged fermions). The $[SU(2)_L]^2 U(1)_Q$ anomaly cancellation condition implies that

$$Q(\nu_e) + Q(\nu_\mu) + Q(\nu_\tau) + 9Q(d) = -3. \tag{6}$$

Thus there are now only two undermined electric charges. One further independent equation can be obtained from the mixed gauge-gravitational anomaly cancellation [9], which says that:

$$Q(\nu_e) + Q(\nu_\mu) + Q(\nu_\tau) = 0. \tag{7}$$

Thus we are left with one undetermined electric charge, which it turns out must be taken as a lepton charge (since eq.(7) and (6) uniquely determine $Q(d) = -1/3$ and hence all the quark charges have been determined). Thus one must conclude that the minimal standard model does not have electric charge quantization. There is one free parameter. Thus an understanding of electric charge quantization requires new physics beyond the minimal standard model. Various ways of extending the standard model so that electric charge is quantized have been discussed in the liturature [1]. One can simply add some terms to the standard model Lagrangian which yield additional constraints. Perhaps the most obvious (and also well motivated) way to do this is to add neutrino masses. For example, one can add three right-handed gauge singlet neutrinos with Dirac and Majorana mass terms. One can assume that there is a non-diagonal CKM type matrix for the leptons which will imply that each generation of leptons have equal charges (just like in the case of the quarks). In addition, Majorana masses for the right-handed neutrinos will fix the charge of the neutrinos to zero. This extension of the standard model would then have every electric charge (ratio) completely determined and hence electric charge quantization would be understood in terms of the internal consistency of the theory.

There is one important point that should be mentioned. The quantum constraints which are the anomaly cancellation equations are not quite as well motivated as the classical constraints. For example, all of the classical constraints seem to be very strong constraints in the sense that we know for certain that the electron has a mass. We know for certain that a coupling of W to u and d and u and s exists etc. Therefore, under the assumption that electric charge is conserved, our conclusions derived from the classical structure of the theory, such as $Q(u) - Q(d) = Q(\nu_e) - Q(e) = 1$ and $Q(d) = Q(s) = Q(b)$ seems to be unchallengable. On the other hand, the anomaly constraints are not definitely true. For example, there could exist a set of "mirror" fermions which have the same gauge quantum numbers as the standard fermions (except that left and right chiralities are interchanged), but are too heavy to be seen yet

in experiment. In this case, there would be *no* nontrivial anomaly cancellation equations. So, we feel that it is an interesting question as to whether gauge theories with $U(1)$ factors, can have electric charge quantization classically i.e. can the classical constraints be sufficient to determine all of the electric charges?

So, if we ignore the constraints from anomalies, then as discussed above there are four classically undertermined parameters in the minimal standard model. One can see from Eq.(4) that two of these parameters are extremely well constrained (to within 10^{-21}), one of them is moderately well constrained (to within 10^{-9}) and one of them, the electric charge of the tau neutrino is not well constrained [10] (note however that there are significant indirect bounds on the electric charge from astrophysics if the tau neutrino has a mass less than about 25 keV [7]). Since it is theoretically possible for the charge of the tau neutrino to be non-zero and since as far as we are aware, a non-zero tau neutrino electric charge has never been searched for in experiments, we propose such an experiment to put to the test the standard assumption that the tau neutrino is neutral.

Of course the minimal standard model may not be complete. It is interesting to look for ways to modify the model so that electric charge is quantized classically. If right-handed neutrinos exist and they have Dirac mass terms with the usual left-handed neutrinos and we assume that nontrivial mixing effects in the weak interaction occur (just like in the quark sector), then in this case, $Q(\nu_e) = Q(\nu_\mu) = Q(\nu_\tau)$, so that there are only two classically undetermined electric charges, which can be taken to be $Q(\nu_e)$ and $Q(d)$. If there is a Majorana mass term for one or more of the right-handed neutrinos then one obtains the additional constraint that $Q(\nu_e) = 0$. Thus, in this case, there is only one undetermined electric charge, which can be taken to be the electric charge of the down quark, $Q(d)$.

Following our philosophy, we need to modify the Lagrangian so that $Q(d)$ is uniquely determined. Another way of thinking about the problem is in terms of global $U(1)$ symmetries. At the classical level, the minimal standard model Lagrangian has four global symmetries: $U(1)_{L_e}, U(1)_{L_\mu}, U(1)_{L_\tau}, U(1)_B$ and one local symmetry $U(1)_Y$. At the classical level, there is no theoretical reason why any combination of Y and L_e, L_μ, L_τ, B cannot be the one $U(1)$ which is gauged. This means that there is a four parameter uncertainty in the $U(1)$ which is gauged. When we modify the lepton sector by adding right-handed (gauge singlet) neutrinos and include mass and mixing terms for the neutrinos, then this new Lagrangian has, in general, only one global symmetry, which is baryon number $U(1)_B$. Hence at the classical level, any combination of Y and B is a $U(1)$ symmetry and can be the $U(1)$ which is gauged. To obtain correct electric charge quantization, we must modify the theory such that baryon number is violated (but with Y left conserved of course). Unlike the case of the lepton sector, we cannot do this by simply adding Majorana mass terms. This works for the leptons since Majorana masses violate the global lepton number (but conserve standard hypercharge). However for quarks, any Majorana mass would

violate both baryon number and standard hypercharge (leaving some linear combination conserved which would consequently not correspond to the electric charges of the real world). Assuming only the standard model gauge symmetry, then the simplest way that we know about to modify the theory to obtain electric charge quantization is to add a new scalar such that its interactions violate the baryon number symmetry (but conserve hypercharge) [11]. The scalar must interact with quarks if it is to violate baryon number. Assuming the usual renormalizable dimension four (Yukawa-type) coupling, then there are only a finite number of possible quantum numbers for the scalar. Since the scalar will couple to a fermion bilinear, it follows from gauge invariance that the quantum numbers of the scalar are those of the fermion bilinears. For example, a scalar σ_1 coupling via the interaction term $\mathcal{L} = \lambda \sigma_1^\dagger \bar{Q}_L (f_L)^c$ implies that σ_1 transforms like $\bar{Q}_L(f_L)^c$. Thus we can simply list the possible scalars in terms of fermion bilinears with $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ representations as follows:

$$
\begin{aligned}
\sigma_1 &\sim \bar{Q}_L (f_L)^c \sim (\bar{3}, 1+3, -y_d) \\
\sigma_2 &\sim \bar{Q}_L e_R \sim \bar{u}_R f_L \sim (\bar{3}, 2, -3-y_d) \\
\sigma_3 &\sim \bar{Q}_L (Q_L)^c \sim (3+\bar{6}, 1+3, -2-2y_d) \\
\sigma_4 &\sim \bar{u}_R (d_R)^c \sim (3+\bar{6}, 1, -2-2y_d) \\
\sigma_5 &\sim \bar{u}_R (e_R)^c \sim \bar{d}_R (\nu_R)^c \sim (\bar{3}, 1, -y_d) \\
\sigma_6 &\sim \bar{u}_R (\nu_R)^c \sim (\bar{3}, 1, -2-y_d) \\
\sigma_7 &\sim \bar{d}_R f_L \sim \bar{Q}_L \nu_R \sim (\bar{3}, 2, -1-y_d) \\
\sigma_8 &\sim \bar{u}_R (u_R)^c \sim (3+\bar{6}, 1, -4-2y_d) \\
\sigma_9 &\sim \bar{d}_R (d_R)^c \sim (3+\bar{6}, 1, -2y_d)
\end{aligned}
\tag{8}
$$

where our notation for the standard model fermions (+ right-handed neutrinos) is as follows:

$$
\begin{gathered}
f_L \sim (1,2,-1), \quad e_R \sim (1,1,-2), \quad \nu_R \sim (1,1,0), \\
Q_L \sim (3,2,1+y_d), \quad u_R \sim (3,1,2+y_d), \quad d_R \sim (3,1,y_d),
\end{gathered}
\tag{9}
$$

with generation index suppressed. We will assume for simplicity that there exists only one exotic scalar. Note that the above interactions do not, by themselves break baryon number since the scalar can carry baryon number. We need to break baryon number in the scalar potential. Note that since all of the scalars are either in the 3 or 6 representation of $SU(3)_c$, the smallest dimensional term which breaks baryon number and conserves $SU(3)_c$ is the trilinear term σ^3. (Note that there is no quadratic or quatic term which breaks baryon number and conserves $SU(3)_c$). Any σ^3 term will also violate standard hypercharge. The only possible renormalizable term must involve 3 sigma's and the Higgs

doublet ϕ. Since the Higgs doublet has hypercharge 1 (in our normalization), a $\sigma^3\phi$ or $\sigma^3\phi^\dagger$ term will imply that the σ scalar must have hypercharge $-1/3$ or $1/3$ respectively. The only candidate for σ is σ_7 since any other choice will clearly lead to the wrong hypercharge assignments. [For example a $\sigma_1^3\phi$ would constrain the hypercharge of σ_1 to be $-1/3$, which will consequently constrain $y_d = -1/3$. Then using eq.(3), we would find that the electric charge of the d quark would by $-1/6$ which would lead to incorrect electric charges for the hadrons, and of course does not correspond to the real world.] Thus, we conclude that under the assumption of only one exotic scalar, electric charge can be quantized classically. Furthermore the quantum numbers of the scalar and the form of the interactions of the scalar are uniquely determined. The scalar couples leptons to quarks through the Lagrangian terms:

$$\mathcal{L} = \lambda_1 \bar{f}_L \sigma d_R + \lambda_2 \bar{Q}_L \sigma^c \nu_R + H.c., \tag{10}$$

where gauge invariance of this Lagrangian term implies that

$$\sigma \sim (\bar{3}, 2, -y_d - 1). \tag{11}$$

The hypercharge of σ is constrained to be $-1/3$ (which means that y_d is constrained to be $-2/3$) by the scalar potential terms:

$$\Delta V(\phi, \sigma) = \lambda \sigma^3 \phi + H.c. \tag{12}$$

Thus, the interactions of σ fix the undertermined hypercharge of the d quark, resulting in a model with electric charge quantization at the classical level. Note that we must choose the parameters in the scalar potential such that σ does not get any VEV, while ϕ of course gets a VEV. It is straightforward to show that this is possible. We leave the details as an exercise to the reader.

Since σ violates baryon number, interactions involving σ will induce baryon number violating processes. The process which should place the most stringent limit on the mass of σ will be the experimental bound on proton decay. Observe that any Feynman diagram leading to proton decay must involve the baryon number violating $\sigma^3\phi$ interaction. The leading order diagram for the proton decay involves one of these interactions, and thus contains three σ fields (note that when the VEV of ϕ is included, the $\sigma^3\phi$ interaction containes a σ^3 interaction term). One can easily see that the simplest diagram giving proton decay leads to the decay $P \to \pi^+ + \nu + \nu + \nu$. The order of magnitude of the decay width for this decay can be evaluated from simple dimensional arguements,

$$\Gamma(P \to \pi^+ + \nu + \nu + \nu) \sim \mathcal{O}\left(\frac{\langle\phi\rangle^2 M_P^{11}}{M_\sigma^{12}}\right), \tag{13}$$

where M_P is the proton mass, and M_σ is the σ scalar mass. Thus, applying the existing experimental limit on the lifetime of the proton we find that the mass of σ is constrained to be greater than about 10^5 GeV.

Finally note that this model may be easily modified so that only the electric charge parameter $Q(\nu_\tau)$ is undetermined. The σ field can be introduced with interactions described above to fix $Q(d) = -1/3$. In the lepton sector it is possible that the third generation does not mix with the first two generations and that the third generation neutrino (i.e. the tau neutrino) is a Dirac fermion (note that a Majorana fermion must have zero electric charge if electromagnetism is unbroken). In this case, the resulting model would have $Q(\nu_\tau)$ classically undertermined. Its mass can be large enough (i.e. greater than about 25 keV [7]) to evaide the astrophysical constraints. Thus, we emphasise again the importance of putting the standard assumption of a electrically neutral ν_τ to the test.

For completeness we mention that a different type of mechanism for obtaining electric charge quantization in a theory with a $U(1)$ gauge factor is possible if the gauge group is enlarged so that a discrete symmetry interchanging the quarks and leptons is assumed. The resulting quark-lepton symmetric models can have a $U(1)$ factor in the gauge group which can be completely fixed classically [12].

In conclusion, we have discussed the issue of electric charge quantization in the standard model. There are two different ways in which the minimal standard model constrains the electric charges of the fermions. There are constraints which follow from the classical structure of the theory and those which follow from the quantization of the theory (i.e. anomaly cancellation). We argue that the classical constraints are very well motivated constraints, while the anomaly conditions are not as well motivated, since new (heavy) fermions could exist which cancel any potential anomalies. We examined the classically allowed electric charges of the fermions in the minimal standard model. We made the observation that the electric charge of the τ neutrino may be non-zero, and that the current experimental constraints on the charge of the τ neutrino seem to be very weak. In fact there may be no experimental searches for a charged τ neutrino at all. If this is the case, then we argue that experiments should be undertaken to test the neutrality of the tau neutrino. We then examined ways in which the standard model could be modified so that the electric charges are quantized correctly classically.

References:

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[2] Note that we have implicitly assumed that $SU(3)_c$ is unbroken and hence each quark colour will have the same electric charge. Thus, there are only two quark charges in each generation.

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[7] In addition to direct experimental bounds obtained from Laboratory experiments, there are in the liturature indirect bounds on the charge of a hypothetical mini-charged particle. In particular, an upper limit of $10^{-13}e$ can be obtained from requiring that plasmon decay into neutrino pairs in red giants be not too efficient [see J. Berstein et al, Phys. Rev. 1963]. This bound is applicable only when the charged particle has mass less than 25 keV. So, if this bound is taken seriously, a tau neutrino can only have a experimentally interesting electric charge if its mass is greater than 25 keV. Also note that an electric charge in the range $10^{-9} \leq Q(\nu_\tau) \leq 10^{-7}$ has been shown to be inconsistent with observations of SN1987A [see R. N. Mohapatra and I. Z. Rothstein, Phys.Lett.B247, 593 (1990); R. N. Mohapatra and S.N. Nussinov, Int. J. Mod. Phys. A8, 3817 (1992).]

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[10] Note that a non-zero value for $Q(\nu_\tau)$ will imply that the charge of the tau ($Q(\tau) = 1 + Q(\nu_\tau)$) will be slightly different to the electron electric charge. However, we expect that the effect of a non-zero tau neutrino charge should be

the most important phenomenologically.

[11] Note that the resulting model was mentioned by a footnote in Foot, Joshi, Lew and Volkas, Ref [1].

[12] R. Foot and H. Lew, Phys. Rev. D41, 3502 (1990); Mod. Phys. Lett. A5, 1345 (1990).